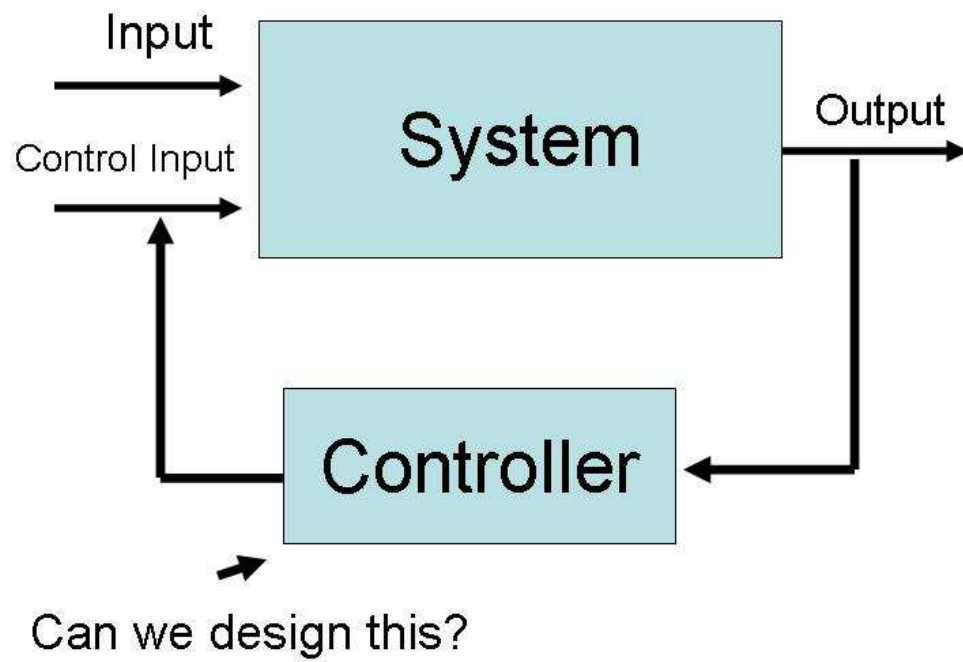


Multivariable Control System

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Chapter 1

General idea about control systems

In this section, we will be dealing mainly with this type of question.

Given: $u(t)$ as input

$x(0)$ as initial condition

We have a system:

$$\dot{x} = f(x, u)$$

$$y = h(x, u)$$

Our goal is to find:

$$x(t), \text{ at } t > 0$$

and

$$y(t)$$

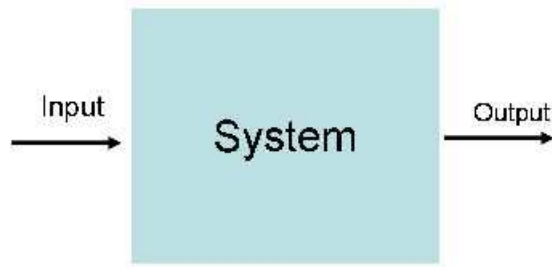
To solve these types of problems, it is cumbersome to do everything by hand. For the purpose of this book, we will use functions in matlab to make your life easier. We will be dealing mostly with these functions.

fft, ode45 , lyap, lqe, lqr

Make sure you check on your matlab for the availability of these functions by using the help function. (ex: help fft) The help menu will give you a general idea of how each of them work. Another tool we will refer to often is simulink. This is a part of matlab. You can type in simulink into your matlab command to see if it is available.

Now that we got that out of the way, let's talk about the type of questions we will be dealing with. When we look at a system, there are two types of problem you will encounter.

The first type is analysis



We look at this system, and ask

is this system stable?

can we control this system? controllable?

can we observe the system? observable ?

Example:

consider the system

$$\dot{x} = Ax + Bu$$

Given:

$x_0, x(0)$, initial state

$x_f, x(\tau)$, final state

$\tau > 0$, time at the final state

Question:

Is there a control input u that could move the system

from state x_0 to x_f in τ second?

This is the type of question you would ask if you are trying to get to the moon. Is it even possible to get there? From solving this type of question you would be able to solve for the exact input you could apply to reach from the starting point x_0 to the destination x_f .

A system is controllable if it is possible to go from any point A to any point B in any finite time T. From this ,we see that going to the moon would not a controllable system since we cannot go from the Earth to the Moon in *any* amount of time. With our technology today, we still can't make it to the moon in less than a second. (too bad)

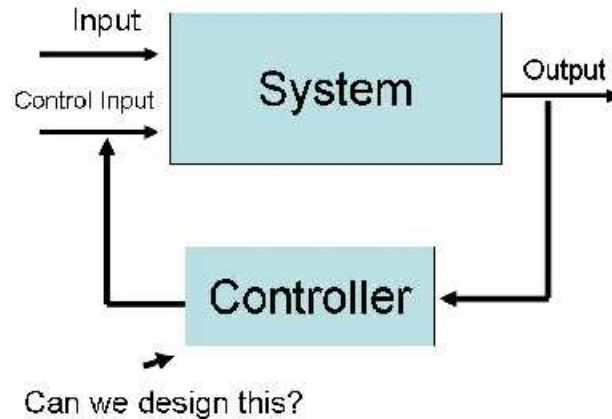
As you can see, it is practically impossible to have a controllable system. You will not likely run into one. We will be dealing with a less strict controllable system throughout the book. We will be looking at the *controllability* of a system.

At this point, it is important to distinguish controllable Vs controllability.

- A system is *controllable* if it meets that practically impossible criteria
- A system's *controllability* tells us if we can use a controller to manipulate output

As you will learn later, every system will have a controllable part and a part that cannot be controlled. We will still be able to control the system if the un-controllable part is “nice.”

The second type of question we face deals with Design.



Now that we have decided that a system is can be controlled, can we design a controller to make it work the way we want?

Given: requirement for the performance of the system

Question: Can we design it?

With a design problem, we use 3 models to characterize the our system.

α . *Real Model*

β . *Controlled Model*

γ . *Evaluation Model*

First is the *real model*, this is normally a very complicated non-linear system. Almost all practical systems will fall into this category. However, due to the complexity we often idealize the model to create the *controlled model*. This idealized model is normally the linearization of the non-linear model. Unfortunately, the linearized model is often not good enough, so we purposely add extra factors into the controlled model, such as friction, resistance, etc.

Chapter 2

Essential Math review

Since all the required math are provided in the previous book “Linear system.” This section will refrain from proving many properties we have seen before.

Characteristic Equations:

If we want to find the determinant of a matrix A, we use the equation

$$\det(\lambda I - A) = 0$$

From this equation, it results in the multiplication of many polynomials. If we generalize the eigenvalues to any value S, we would get the characteristic equation for matrix A.

$$\det(SI - A) = 0$$

We will denote the characteristic equations of matrix A as $\text{char}(A)$ in this book. The general form for $\text{char}(A)$ is then:

$$S^n + \alpha_{n-1}S^{n-1} + \alpha_{n-2}S^{n-2} + \dots + \alpha_0 = 0$$

Cayley Hamilton Thm:

According to the Cayley Hamilton theorem, if we have $\text{char}(A)$ such that

$$S^n + \alpha_{n-1}S^{n-1} + \alpha_{n-2}S^{n-2} + \dots + \alpha_0 = 0$$

We can replace the matrix A with S such that:

$$A^n + \alpha_{n-1}A^{n-1} + \alpha_{n-2}A^{n-2} + \dots + \alpha_0 = 0$$

or it could be written as:

$$A^n = -\alpha_{n-1}A^{n-1} - \alpha_{n-2}A^{n-2} - \dots - \alpha_0$$

If we look at the equation above, the Cayley Hamilton Theorem says that a matrix of any power could be represented as a linear combination of smaller power of A.

At this point, it is easy to look at this equation and conclude that you would need all the power up to $n-1$ of A to reconstruct A^n . Since the equation shows the linear combination of all powers of A up to $n-1$, this would be a logical conclusion. However, if you notice that A to the power of $n-1$ is also a power of A. Therefore, A to the power of $n-1$ could also be represented as a power lower than $n-1$.

To make the point more clear:

A matrix taken to a power of N can be represented by a linear combination of ANY lower powers of N except 0.

Taylor Expansion of an Exponential

If we let $A \in \mathbb{C}^{n \times n}$ then

$$e^{At} = I + At + \frac{(At)^2}{2!} + \frac{(At)^3}{3!} + \frac{(At)^4}{4!} + \dots$$